Anisotropic propagation speed of light on rotating Earth's surface - theoretical derivation, implications for definition of meter, and crucial experiment

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Abstract

Whether the propagation speed of light at the surface of rotating Earth is isotropic or not is both an important theoretical problem and one that has significant implications for scientific practices. Especially the unit of length, a meter, is defined as the length of the path travelled by light in vacuum in $1/299792458$ of a second, on the basis that the speed of light is a constant, c = 299792458 m/s. By applying the GPS range equation whose correctness has been fully verified by GPS practices, we found that the propagation speed of light on rotating Earth's surface is neither a constant nor isotropic, but $c' = c - \boldsymbol{v}_{rE} \cdot \hat{\mathbf{d}}$, where $\boldsymbol{v}_{rE}$ is the local linear velocity of Earth's rotation, and $\hat{\mathbf{d}}$ is the unit vector of the light propagation's direction. It follows that the anisotropy of the propagation speed of light at the Earth's surface has a significant impact on the definition of the meter. Based on this finding, and the difference between Earth rotation's Sagnac effects on the equator and the meridian, we propose a crucial experiment to examine the anisotropy of the propagation speed of light: let a stable pulsed laser emitting pulses in two opposite directions, use the ultrafast imaging techniques for visualizing, measuring and comparing the spacing of the pulses in opposite directions. Then we can decisively conclude whether the speed of light is anisotropic or not.

Keywords: Anisotropic speed of light; Rotating Earth; Theoretical derivation; Definition of meter; Crucial experiment

I. Introduction.

According to the theory of special relativity [1], in any inertial system, the one-way speed of light in vacuum is constant and isotropic. However, an interesting reality is that although it is widely accepted that the Earth rotates around its axis from west to east and that the Earth system is not an inertial system, the vast majority of people believe that in the Earth system, especially on the Earth's surface where experiments are easier to perform, the one-way speed of light in vacuum is also constant and isotropic.

Then why would the well-known fact that the Earth is rotating and the Earth system is a non-inertial system not affect most people's judgments about the speed and isotropy of light in the Earth system, including the speed and isotropy of light propagating on the Earth's surface? There were just a number of experiments [2-5] that claimed to show, with a high degree of precision, that the propagation speed of light on the Earth's surface is isotropic. However, a closer analysis of these experiments reveals that they have one thing in common: the light beam is reflected back and forth many, many times, meaning that no experiment has ever really tested whether the one-way propagation speed of light is isotropic on the rotating Earth.

Therefore, the claim that the propagation speed of light at the surface of the rotating Earth is isotropic has neither a theoretical basis nor experimental verification. On the other hand, similar to the Sagnac experiment [6, 7] which shows on a rotating disk, that the clockwise and counterclockwise light beams take different time intervals to travel the same closed path, Michelson and Gale [8, 9] built a large interferometer to show that the Earth rotation also causes two counter-propagating light beams to show a travel-time difference. The Sagnac effect caused by rotation is the basis for fiber optic gyroscopes and ring laser gyroscopes that measure all rotations, including the rotation of the Earth [10-14]. And in the two-way satellite time and frequency transfer (TWSTFT) using the propagation of electromagnetic waves among satellites and ground stations, we can clearly find the directionality of the Sagnac effect caused by the Earth's rotation [15-18].

In this paper, we analyze the propagation of light between two adjacent points on the Earth's surface using the GPS range equation. We found that the propagation speed of light on Earth's surface is neither constant nor isotropic. The propagation speed of light is $c' = c - v_{rE}\cos\theta$, where v_{rE} is the local linear velocity of rotation of the Earth, and θ is the angle between v_{rE} and the direction of light propagation.

We also discuss the implication of the anisotropy of the propagation speed of light on the definition of the meter. The unit of length, meter, is defined as the length of the path travelled by light in a vacuum in 1/299792458 of a second. Obviously, this definition is based on that the speed of light is exactly a constant regardless time, location, or direction. However, the propagation speed of light on rotating Earth's surface being anisotropic destroys the foundation of this definition. For example, on the equator, the eastward speed of light is 932 m/s slower than the westward speed. Therefore, in 1/299792458 of a second, the length of the eastward light path is 3.1 μm shorter than the westward's. Obviously, this difference cannot be neglected, and is also completely detectable.

Based on these, and the difference between Earth rotation's Sagnac effects on the equator and on the meridian, we propose a crucial experiment to examine the anisotropy of the speed of light which would allow us to decisively conclude whether $c' \equiv c$ or $c' = c - \boldsymbol{v}_{rE} \cdot \hat{\mathbf{d}}$.

II. Theoretical derivation.

In the operations of GPS, the equation that depicts the governing law of electromagnectic wave propagation near the Earth is the range equation in an Earth-centered inertial (ECI) system [19-21]:

$$\left|\boldsymbol{r}_r(t_r) - \boldsymbol{r}_s(t_s)\right| = c(t_r - t_s)\,, \tag{1}$$

where t_s is the instant of signal transmission from the source, and t_r is the instant of reception at the receiver; $\boldsymbol{r}_s(t_s)$ is the position vector of the source at the transmission time, and $\boldsymbol{r}_r(t_r)$ is the position vector of the receiver at the reception time. The correctness of the GPS range equation has been fully proven by GPS practices. Now we use GPS range equation to analyze the propagation of light waves near the Earth and specifically to analyze the light propagation from point A, a source, to another co-moving point B , a receiver.

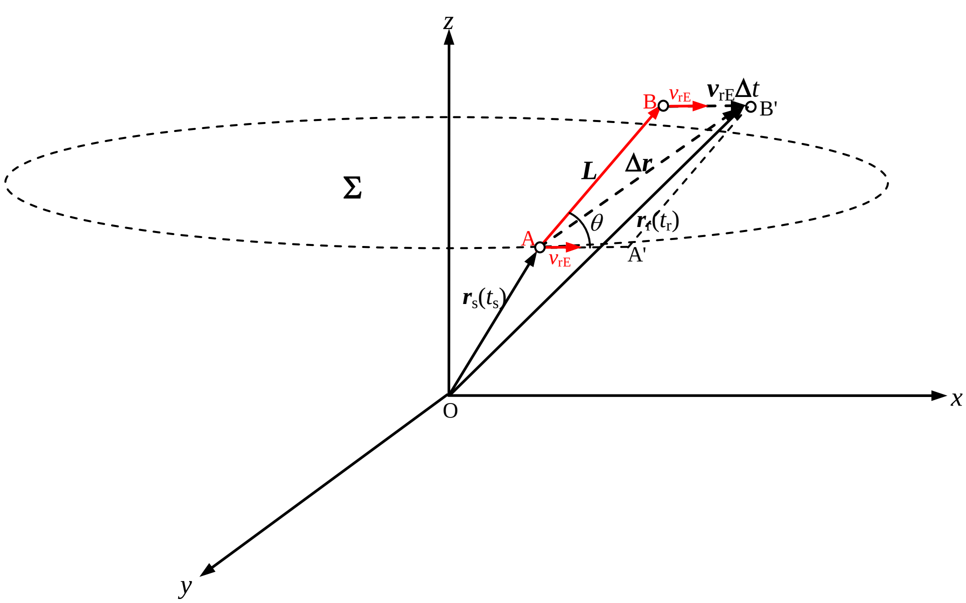

Fig. 1. A light wave propagates from A to B and both A and B are co-moving with the same velocity $\boldsymbol{v}_{\mathrm{rE}}$.

Assuming in an Earth-centered inertial system Σ, there are two fixed points A and B on the Earth surface (Fig. 1), and their distance L is not large, so both A and B have the same linear velocity of rotation from west to east $\boldsymbol{v}_{\mathrm{rE}}$, and the angle between the vectors $\boldsymbol{L}$ and $\boldsymbol{v}_{\mathrm{rE}}$ is θ. Let's calculate what the propagation speed of light is when a beam is sent from A to B.

There the propagation of light from A to B becomes what is presented in the figure: the light starts from point A and the vector from the origin O of Σ to point A is $\boldsymbol{r}_s(t_s)$. And while the light reaches the receiving point after a time interval Δt, since B

is moving in Σ, B reaches point B' and $BB' = v_{rE}\Delta t$, and therefore the vector from the origin O to B' is $\boldsymbol{r}_r(t_r)$. Obviously $(t_r - t_s)$ in the GPS range equation is the Δt we mentioned here.

In the figure, there is a vector triangle, $\boldsymbol{L}$, $\boldsymbol{v}_{rE}\Delta t$ and $\Delta\boldsymbol{r}$ and $\Delta\boldsymbol{r} = |\boldsymbol{r}_r(t_r) - \boldsymbol{r}_s(t_s)|$. According to Eq. (1), we have $\Delta\boldsymbol{r} = c\Delta t$.

Based on the relationship between the three vectors $\boldsymbol{L}$, $\boldsymbol{v}_{rE}\Delta t$ and $\Delta\boldsymbol{r}$, we have

$$|\Delta\boldsymbol{r}|^2 = L^2 + (v_{rE}\Delta t)^2 + 2\boldsymbol{L}\cdot\boldsymbol{v}_{rE}\Delta t\,. \tag{2}$$

That is $c^2\Delta t^2 = L^2 + (v_{rE})^2\Delta t^2 + 2\boldsymbol{L}\cdot\boldsymbol{v}_{rE}\Delta t$.

Because of $(v_{rE})^2 << c^2$, then,

$$\Delta t^2 - (2\boldsymbol{v}_{rE}\cdot\boldsymbol{L}/c^2)\Delta t - L^2/c^2 = 0\,. \tag{3}$$

Solving this equation for Δt yields

$$\Delta t = L/c + \boldsymbol{v}_{rE}\cdot\boldsymbol{L}/c^2\,. \tag{4}$$

That means the propagation time of this beam from A to B over the length L is

$$\Delta t = L/c + \boldsymbol{v}_{rE}\cdot\boldsymbol{L}/c^2 = L/c + v_{rE}L\cos\theta/c^2\,.$$

Now we define c' as the speed of light of this ray, i.e., $c' = L/\Delta t$, then we have

$$c' = L/(L/c + v_{rE}L\cos\theta/c^2) = c^2/(c + v_{rE}\cos\theta) \approx c - v_{rE}\cos\theta\,, \tag{5}$$

neglecting the quantities of second and higher orders of (v_{rE}/c) . There, v_{rE} is the local linear velocity of rotation of the Earth, and θ is the angle between $\boldsymbol{v}_{rE}$ and the direction of light propagation.

The expression $c' = c - v_{rE}\cos\theta$ can also be presented in the form

$$c' = c - \boldsymbol{v}_{rE}\cdot\hat{\mathbf{d}}\,, \tag{6}$$

where $\hat{\mathbf{d}}$ is the unit vector of the direction of light propagation.

From Fig. 1 we can find that regardless of which direction $\boldsymbol{L}$ is in, the required solution is always a vector triangle consisting of $\boldsymbol{L}$, $\boldsymbol{v}_{rE}\Delta t$ and $\Delta\boldsymbol{r}$ (= $c\Delta t$, according to Eq. 1), and therefore the calculated relation among Δt and $\boldsymbol{L}$, $\boldsymbol{v}_{rE}$, c is the same. Since $c' = L/\Delta t$, we finally have that the relationship among c' and c, $\boldsymbol{v}_{rE}$, $\hat{\mathbf{d}}$ is always the same.

The difference between this result $c'/c = 1 - v_{rE}\cos\theta/c$ and general belief that the propagation speed of light at the Earth's surface is isotropic, i.e., $c'/c \equiv 1$, is a first-order quantity, $v_{rE}\cos\theta/c$, so this result is not influenced by Lorentz contraction and relativistic time dilation, the second-order quantities [$\propto (v_{rE}/c)^2$].

III. The implications of the anisotropic propagation speed of light on the definition of a meter, the unit of length.

Let us discuss a very important implication in physics caused by the anisotropy of the propagation speed of light on the Earth's surface, the implications of the anisotropic propagation speed of light on the definition of a meter.

Length is the first fundamental quantity in physics. It is well known that since 1983 the unit of length, a meter, is defined as the length of the path travelled by light in vacuum during a time interval of 1/299792458 of a second. Obviously, this definition requires that the speed of light in a vacuum be exactly the constant, 299792458 m/s, at anytime, anywhere, and in any direction. However, as we know from the theoretical analysis in the previous section, on the Earth surface where we primarily use the definition of a meter, the propagation speed of light is not a constant, and it is anisotropic. This apparently destroys the basis of the definition of the meter, thus making the important fundamental quantity in physics - the length, one that bears the errors.

Let us make the problem more specific: since a meter is defined that way, we can make a light ruler similar to a tape measure: a pulsed laser emits a beam with a pulse period of 1/299792458 seconds and an ultrashort pulse duration, e.g., 0.3 ps. Then the distance of the spatial separation between two pulses is 1 meter, and we call it light pulse ruler (LPR). As a comparison,

when we measure a distance with a tape measure, we pull the tape and observe how many meter mark intervals the distance fits and thus know how many meters the distance is. Similarly, when we measure a distance with an LPR, we turn on the LPR and check how many pulse intervals the distance fits and then know how many meters the distance is.

Now we will use the LPR as a tool to measure a distance at the equator. Obviously, to do this, just as we are sure that the length of one meter on a tape measure is indeed one meter, no matter which direction we measure, we also have to be sure that the spacing between two pulses of the LPR is always 1 meter, no matter which direction the pulses are directed. But can we quite be sure of this? Actually, from the analysis in the previous section, we know that the speed of light at the equator is not isotropic; eastward is c - 466 m/s, westward is c + 466 m/s (the linear velocity of the Earth's rotation at the equator v_{rE0} = 466 m/s), and southward and northward are both c. Since the distance between two pulses is the pulse period multiplied by the speed of light at this place and in this direction, for the spacing between two pulses in different directions, although we would like to have a circle with $\rho_0 = 1\,\text{m}$, we will actually obtain a graph of $\rho_1 = (1 - a\cos\theta)$ m, where a = 466/299792458. In particular, the difference between LPR pointing to the east and pointing to the west amounts to $\Delta\rho = 3.1\,\mu\text{m}$. In other words, the eastward one is 3.1 μm shorter than the westward one. This means when we use such an LPR to measure a certain distance on the equator, for a meter, the difference between measuring to the east and measuring to the west reaches 3.1 μm. This is an error that even cannot be allowed in precision machinery manufacturing and one that can be detected without much difficulty. Moreover, at a latitude of 45 degrees, the difference between the eastward and westward measurements with such an LPR is $\cos 45^0$x3.1 μm = 2.2 μm per meter.

We can also notice that if we compare the two directions, south and north, the spacing between the pulses of an LPR sent south is the same as the spacing between the pulses sent north.

In fact, these also give us indications of how to conduct experiments on the anisotropy of the speed of light at the Earth's surface.

IV. Insights derived from the Sagnac experiment and Michelson-Gale experiment.

We now apply the well-known Sagnac effects from the Sagnac and Michelson-Gale experiments to the simple but very large equatorial and meridional fiber Sagnac interferometer to find out what happens there and what insights they give us regarding how to conduct experiments.

1, Wrap a vacuumed air-core fiber around the equator to make a large fiber Sagnac interferometer (Fig. 2).

2, When we use continuous waves to conduct the Sagnac experiment, due to the Sagnac effect caused by the Earth's rotation, we can observe that although both light beams are around the Earth's equator one turn, $L_{EQ} = 40075 \cdot 10^3$ m, the beam propagating in the same direction of the Earth's rotation takes a longer time than that the beam in the opposite direction takes, and the time difference between the two, the Sagnac effect, is $\Delta t = t_E - t_W = 2v_{rE0}L_{EQ}/c^2 = 415.58$ ns.

3, Now we use pulsed laser to conduct this Sagnac experiment, and choose a pulse period of 1/299792458 sec and a pulse duration of 0.3 ps.

If we choose to look closely at a pulse P_A that departs from the beam splitter O at a certain time, it first splits into two, P_{AE} emitting eastward and P_{AW} emitting westward, and then each goes around the fiber loop with a length of L_{EQ} and returns back to the origin O. It can be observed that due to the Sagnac effect, when P_{AW} returns to the origin, P_{AE} is not there yet, but arrives 415.58 ns later. This amount of delay can also be expressed as the difference in the number of pulses: $\Delta n = 124.58$.

That is, the number of pulses n_E in the eastward optical path over the total length L_{EQ} is more than the number of pulses n_w in the westward optical path by $\Delta n = n_E - n_W = 124.58$.

And if the Earth does not rotate, the number of pulses, n_o, in the equatorial circumference L_{EQ} is $n_0 = 40075 \cdot 10^3$. Thus, we have that due to the rotation of the Earth to the east, the ratio of Δn with respect to n_0, the original total number of pulses, is $\Delta n / n_0 = 124.58 / 40075 \cdot 10^3 = 3.1086 \cdot 10^{-6}$.

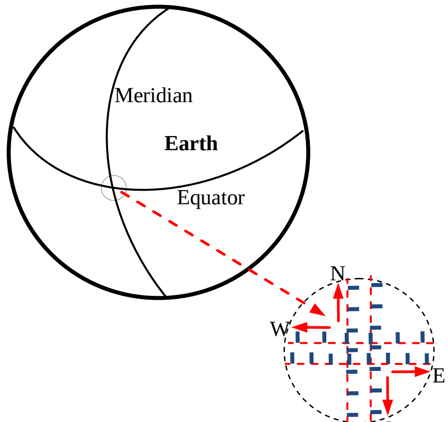

Fig. 2. Equatorial fiber Sagnac interferometer and meridional fiber Sagnac interferometer

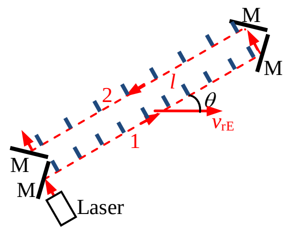

Fig. 3. Crucial experiment examining the anisotropy of the speed of light on the rotating Earth's surface. M - Mirror.

4, Since the total length of both light paths is the same L_{EQ}, the increase in the number of pulses in the eastward light path compared to the westward light path definitely means that the pulse spacing of the eastward light path ρ_E is smaller than that of the westward light path ρ_W, and we have $(\rho_W - \rho_E)/\rho_0 = 3.1086 \cdot 10^{-6}$.

For a pulse spacing of about 1 m, the eastward pulse is 3.1 μm shorter than the westward pulse. This is the difference we want to measure, and the one we are able to measure. This is also exactly the same as the result we deduced above after deriving the anisotropy of the speed of light at the Earth's surface $\boldsymbol{c}' = \boldsymbol{c} - \boldsymbol{v}_{rE} \cdot \hat{\mathbf{d}}$ based on the GPS range equation.

5, Now we build a fiber-optic Sagnac interferometer on one of the meridians, and do the similar thing as the equatorial Sagnac interferometer, i.e., sending pulses to the south and north. Since the rotation of the Earth has no effect on the meridional Sagnac interferometer, we can conclude that the results are $n_S = n_N = n_0$, and $\rho_S = \rho_N = \rho_0$.

6, If we take a very fast photograph at the junction of the two fiber optic interferometers, we will have this image: the eastward and westward pulsed beams have different pulse spacing, and the eastward one is shorter than the westward one; while the southward and northward pulsed beams have the same pulse spacing (see the enlarged view on the corner of Fig 2). This is a good indication of how to design the crucial experiment.

V. Crucial experiment examining the anisotropy of the speed of light on the rotating Earth's surface.

In a laboratory with latitude ϕ_A, a stable pulsed laser with visible light is chosen for the LPR, which has the parameters of pulse period 1/(10x299792458) s and pulse duration 0.03 ps. Construct the LPR with the following configuration: there are two beams in opposite directions, and l being 1.5 m, which means that there are always more than a dozen pulses in the two directions (Fig. 3).

Choose a directional angle θ. As shown in Fig. 3, we place this LPR main axis at an angle θ to the direction of local Earth rotation. At present, single-shot ultrafast imaging techniques have achieved the imaging frame rates of 70 to 256 trillion frames per second and can be used on the visualization and measurement of the propagation of light pulses [22, 23]. Therefore, with these techniques we can measure the lengths of each ten pulses on the two optical paths 1 and 2, ρ_1 and ρ_2, and find $\rho_2 - \rho_1$.

According to the calculation above we should have $\rho_1 = \rho_0(1 - v_{rE}\cos\theta / c)$, and $\rho_2 = \rho_0(1 + v_{rE}\cos\theta / c)$.

Hence, $\rho_2 + \rho_1 = 2\rho_0$, and $\rho_2 - \rho_1 = 2\rho_0 v_{rE}\cos\theta / c = 2\rho_0 v_{rE0}\cos\phi_A \cos\theta / c$.

Then,

$$(\rho_2 - \rho_1)/(\rho_2 + \rho_1) = v_{rE}\cos\theta / c = v_{rE0}\cos\phi_A \cos\theta / c . \qquad (7)$$

Therefore, if the experimental result is $(\rho_2-\rho_1)/(\rho_2+\rho_1)=0$, the propagation speed of light at the Earth's surface is isotropic; and if the result is $(\rho_2-\rho_1)/(\rho_2+\rho_1)=v_{rE}\cos\theta/c=v_{rE0}\cos\phi_A\cos\theta/c$, it proves that the propagation speed of light at the Earth's surface is anisotropic and $c'=c-v_{rE}\cos\theta=c-v_{rE0}\cos\phi_A\cos\theta$.

This crucial experiment is a first-order experiment (the result is proportional to *v*/*c*) and, as we calculated earlier, at latitudes that are not particularly high, $(\rho_2-\rho_1)/(\rho_2+\rho_1)$ is a quantity that can be detected relatively easily if it exists.

VI. Conclusions.

Whether the propagation speed of light at the surface of rotating Earth is isotropic or not is a very important scientific problem. First, based on the GPS range equation, we theoretically found that the propagation speed of light on the Earth's surface is neither constant nor isotropic, but $c'=c-\boldsymbol{v}_{rE}\cdot\hat{\mathbf{d}}$. We also discussed its implication on the definition of the unit of length, a meter. Finally, we presented a feasible and crucial experiment: let a stable laser emitting ultrashort pulses in two opposite directions, use ultrafast imaging techniques on visualization and measurement of the propagation of pulses, compare the spacing of the pulses in opposite directions. It is hoped that this important problem will be answered by the crucial experiment in the near future.

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